

The Charm and Bottom Hyperons in a Chiral Quark Model

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Abstract

The spectrum of the $C = 1$ hyperons is well described by the constituent quark model, if the fine structure interaction between the light and strange quarks is mediated by the $SU(3)_F$ octet of light pseudoscalar mesons, which are the Goldstone bosons of the hidden approximate chiral symmetry of QCD. With the addition of a phenomenological flavor exchange interaction of the same form between the light and the charm quarks to describe the $\Sigma_c - \Sigma_c^*$ and $\Xi_c^s - \Xi_c^{*s}$ splittings, the splittings between the $C = 1$ states fall within 10-30 MeV of the empirical values. Predictions are presented for the lowest negative parity excited states and the magnetic moments as well. Corresponding predictions for the $B = -1$ hyperon states are also given.

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1. Introduction

A very satisfactory description of the whole observed part of the spectrum of the nucleon, Δ -resonance and the strange hyperons can be achieved with the constituent quark model if in addition to an effective harmonic confining interaction the constituent quarks are assumed to interact by exchange of the $SU(3)_F$ octet of light pseudoscalar mesons, which are the Goldstone bosons of the hidden realization of the approximate chiral symmetry of QCD [1,2]. We here show that the extension of this model to the ground state spectrum of the $C = 1$ charm hyperons predicts a spectrum in good agreement with the empirically known states if it is augmented by a weaker phenomenological heavy flavor exchange interaction that acts between the light and the charm and the strange and the charm quarks respectively. This flavor exchange interaction may be viewed as arising from exchange of D (and D^*) mesons (or systems with the same quantum numbers) between the u, d and the c -quarks and of D_s (and D_s^*) mesons between the s and c quarks. Such flavor exchange interactions requires complete antisymmetrization of the 3 quark states, even for the quarks of widely different mass. This implies a close formal correspondence between the symmetry (and notation) for the $C = +1$ and $S = -1$ hyperon states [3,4].

The mean energy of the 2 lowest excited Λ_c^+ states is predicted to be in good agreement with the empirical value, under the assumption that these states are the charm analogs of the strange flavor singlet $\Lambda(1405) - \Lambda(1520)$ negative parity resonances. Together with the satisfactory prediction of the energies of the $C = +1$ hyperons in the ground state band this indicates that the chiral constituent quark model of refs.[1,2] has the proper heavy quark limit. The predicted spectrum of the ground state bottom $B = -1$ hyperons is very similar to that of the $C = +1$ charm hyperons. Predictions are given for the lowest ("P-shell") negative parity states as well as for the magnetic moments, with inclusion of the small exchange current corrections.

The spin-spin component of the $SU(3)_F$ pseudoscalar octet exchange interaction has the form

$$\begin{aligned}
H_\chi = - \sum_{i < j} \{ & \sum_{a=1}^3 V_\pi(r_{ij}) \lambda_i^a \lambda_j^a + \sum_{a=4}^7 V_K(r_{ij}) \lambda_i^a \lambda_j^a \\
& + V_\eta(r_{ij}) \lambda_i^8 \lambda_j^8 \} \vec{\sigma}_i \cdot \vec{\sigma}_j,
\end{aligned} \tag{1.1}$$

where the radial functions V_π , V_K and V_η represent the π , K and η -exchange interactions respectively, and which have the usual Yukawa form at long range. As the behavior of these functions at short range is not known, their matrix elements in the lowest shells of the harmonic oscillator basis were extracted from the lowest splittings of the nucleon spectrum in ref. [2].

With those matrix elements the spectra of the light and strange baryons were predicted to be in remarkably good agreement with the empirical spectra. Moreover the chiral boson exchange interaction (1) leads to the correct ordering of the positive and negative parity states in the spectra in the spectrum and in contrast to the commonly employed gluon exchange model [5]. The motivation for the interaction (1) is the unique role that the light pseudoscalar octet mesons have as Goldstone bosons of the spontaneously broken approximate chiral symmetry of QCD. Because of their large masses the corresponding charm and charm-strange pseudoscalar mesons D and D_s cannot on the other hand be given any such interpretation. In fact by the near degeneracy between them and the corresponding vector mesons D^* and D_s^* a meson exchange interaction of the form (1) would be expected to be due to both pseudoscalar and vector meson exchange. A phenomenological interaction that represents these types of interaction mechanisms in pairs involving one light and one heavy quark would have the form

$$\begin{aligned}
H_h = - \sum_{i < j} \{ & \sum_{a=9}^{12} V_D(r_{ij}) \lambda_i^a \lambda_j^a \\
& + \sum_{a=13}^{14} V_{D_s}(r_{ij}) \lambda_i^a \lambda_j^a \} \vec{\sigma}_i \cdot \vec{\sigma}_j.
\end{aligned} \tag{1.2}$$

Here the λ_i^a matrices are the $SU(4)$ extension of the $SU(3)_F$ Gell-Mann matrices in (1). As the quark content of the η_c and the J/ψ are purely $c\bar{c}$ these mesons mediate no interaction between the light and charm quarks, and are therefore omitted from (1.2). It will be shown below that an interaction of this form is required to explain the nonvanishing $\Sigma_c^* - \Sigma_c$ and

$\Xi_c^* - \Xi_c^s$ splittings. The interactions V_D and V_{D_s} will be treated as purely phenomenological here, and in fact we shall only need their S -state matrix elements in the oscillator basis.

As the interaction (1) does not act in quark pair states that include charm or bottom quarks, the consequence is that the fine structure of the $C = +1$ and $B = -1$ hyperons should be mainly determined by the interaction (1) in the quark pair with only light and/or strange quarks. As moreover the ground state band is insensitive to the details of the confining interaction, this implies that the energies of the ground state charm and bottom flavor hyperons may, to a first approximation, be predicted without any additional parameters.

This paper is divided into 7 sections. In section 2 we review the harmonic oscillator basis states for the hyperons with one heavy quark, treating the quark mass difference as a perturbation. In section 3 we construct the ground state spectra of the charm and bottom hyperons using the chiral field interaction (1.1) and discuss the role of the flavor exchange interactions between light and heavy quarks. In section 4 we discuss the lowest excited negative parity states of the charm hyperons. Section 5 contains the results for the ground and $L = 1$ state bands of the $B = -1$ hyperons. In section 6 we give predictions for the magnetic moments of the ground state $C = +1$ and $B = -1$ hyperons. Section 7 contains a summarizing discussion.

2. The Basis States for the Heavy Flavor Hyperons

There is no a priori reason to exclude two-body flavor exchange interactions between constituent quarks. Such are automatically implied by any direct quark couplings to the light pseudoscalar mesons, once the meson degrees of freedom are integrated out of the corresponding Fock space. The importance of such couplings appears increasingly compelling [2,6,7,8]. An immediate consequence of flavor exchange interactions is that such imply the necessity of complete antisymmetrization of the 3 quark states that form the baryons even when the 3 quarks have different flavors and constituent masses. This is most readily illustrated by an example. Consider the Λ^0 , the

quark content of which is uds . By definition the u, d quark pair state has to be completely antisymmetric. Kaon exchange between say the u and the s quarks will exchange their flavor coordinates thus leading to an antisymmetric d, s pair state. Iteration of the argument implies that the three quarks have to be in antisymmetric flavor states, and as a corollary that the uds state is necessarily totally antisymmetric. The argument generalizes immediately to the Λ_c^+ , which has the quark content udc . In the latter case the flavor exchange interaction could be mediated e.g. by the D or D^* mesons.

The requirement of the total antisymmetry of the 3-quark wavefunctions, independently of the quark flavors, is met by the $SU(3)$ flavor-spin basis for the charm quarks, which can be constructed directly from the corresponding 3-quark states for the strange hyperons [3,9] by replacing the s quark with the c quark in the Λ and Σ flavor states in the case of the Λ_c^+ and the Σ_c , by replacing either the u and the s or the d and the s quark in the Λ and Σ flavor states by the s and c quarks respectively in case of the Ξ_c^a and Ξ_c^s and finally by replacing the light quark by the c quark in the Ξ wavefunctions in the case of the Ω_c^0 . The only new feature in the case of the $C = +1$ and $B = -1$ hyperons is the appearance of the states Ξ_c^a and Ξ_b^a in which the light and strange quarks form an antisymmetrical combination, as well as the appearance of spin $1/2$ Ω_c^0 and Ω_b^- states in addition to the spin $3/2$ Ω_c^{0*} and Ω_b^{-*} states, which have the same flavor-spin symmetry as the Ω^- [3,4].

We shall describe the effective confining interaction by a harmonic oscillator interaction with flavor independent string tension. The harmonic oscillator Hamiltonian for the 3-quark system is then

$$H_0 = \sum_{i>1}^3 \frac{\vec{p}_i^2}{2m_i} - \frac{\vec{P}_{cm}^2}{2M} + \frac{1}{6} \sum_{i<j} k(\vec{r}_i - \vec{r}_j)^2, \quad (2.1)$$

where M is the sum of the 3 quark masses ($\sum_{i=1}^3 m_i$), and \vec{P}_{cm} is their total momentum. In the case of the $C = 1$ or $B = -1$ hyperons two of the quark masses represent the constituent masses of the light or strange quarks (m) and one that of a heavy flavor quark (m_h).

Because of the mass difference between the light and heavy quarks, the antisymmetrization of the 3 quark wavefunction will lead to a mixing of the

orbital and flavor states so that the hyperon states will lack definite orbital and flavor symmetry. In view of the fact that the harmonic oscillator potential represents but a crude effective representation of the confining interaction we shall here avoid the diagonalization of the oscillator Hamiltonian in the antisymmetric basis with mixed orbital and flavor symmetry and be content to treat the quark mass difference as a flavor dependent perturbation to the equal mass model Hamiltonian:

$$H'_0 = \sum_{i>1}^3 \frac{\vec{p}_i^2}{2m} - \frac{\vec{P}_{cm}^2}{6m} + \frac{1}{6} \sum_{i<j} m\omega^2 (\vec{r}_i - \vec{r}_j)^2, \quad (2.2)$$

where $\omega = \sqrt{k/m}$. The perturbation that arises from the quark mass difference is then

$$H''_0 = - \sum_{i=1}^3 \frac{m_h - m}{2m} \left\{ \frac{\vec{p}_i^2}{m_h} - \frac{\vec{P}_{cm}^2}{3(2m + m_h)} \right\} \delta_{ih}. \quad (2.3)$$

Here the Kronecker δ_{ih} indicates that the perturbation acts only when i equals the coordinate label of the heavy quark. If the term that contains the center-of-mass momentum \vec{P}_{cm} is dropped in (2.3) the only effect of the perturbation (2.3) on the oscillator states in the ground state band will be a lowering of their energies by

$$< g.s. | H''_0 | g.s. > = -\frac{1}{2} \delta, \quad (2.4)$$

where

$$\delta = (1 - m/m_h) \omega. \quad (2.5)$$

The flavor-spin symmetries of the hyperons in the ground state band are $[3]_{FS}[21]_F[21]_S$ and $[3]_{FS}[3]_F[3]_S$, where $[f]_i$ denotes a Young pattern with f being the sequence of integers that indicates the number of boxes in the successive rows of the Young pattern. This should then be combined with the totally symmetric orbital state ($[3]_X$) and the antisymmetric color state ($[111]_C$).

The flavor-spin symmetries of the zero order oscillator wavefunctions of the lowest lying negative parity Λ_c^+ (Λ_b^0) states in the P -shell will be

$[21]_{FS}[111]_F[21]_S$, $[21]_{FS}[21]_F[21]_S$ and $[21]_{FS}[21]_F[3]_S$ respectively. The corresponding zero order wavefunctions of the $\Sigma_c(\Sigma_b)$ states will have the flavor-spin symmetries $[21]_{FS}[21]_F[21]_S$, $[21]_{FS}[3]_F[21]_S$ and $[21]_{FS}[21]_F[3]_S$. The levels of all of these states, which are degenerate at zero order (2.2), will be split by the chiral field interaction (1.1), the heavy flavor exchange interaction (1.2), as well as by the perturbation (2.3) that arises from the quark mass difference. The flavor-spin symmetry of the predicted negative parity resonances of the Ξ_c^a and the Ξ_c^s are the same as these of the Λ_c and the Σ_c above. The states and their symmetry assignments are listed in Tables 1-3.

3. Fine Structure Splitting of the $C = 1$ Ground State Hyperons.

The spectrum of the ground state $C = 1$ charm hyperons is obtained by treating the flavor exchange interactions (1.1) and (1.2) in first order perturbation theory. The correction to the unperturbed level is then expressed in terms of S-state matrix elements of the potential functions V_π, V_K and V_η in the pseudoscalar octet exchange interaction (1.1), as well as of the charm and charm-strangeness exchange potentials in H_H (1.2). We shall denote these matrix elements $P_{nl}^f = \langle nlm | V | nlm \rangle$, where the superscript f indicates the type of flavor exchange. Here $|nlm\rangle$ are 3-dimensional harmonic oscillator wavefunctions. In the case of η exchange the constituent masses of the quarks in the interacting pair is indicated explicitly (here $m_u = m_d$). The fine structure corrections to the different hyperon states are expressed in terms of such integrals in Table 1. In these fine structure corrections we have also included the difference Δ_s between the constituent masses of the light u, d and s quarks, as well as the energy shift $-\frac{1}{2}\delta$ (2.4) that is caused by the quark mass difference.

The pion and K exchange matrix elements P_{00}^π and P_{00}^K were extracted from the empirical $\Delta_{33} - N$ and $\Sigma(1385) - \Sigma$ mass differences to be 29.05 MeV and 20.1 MeV in ref. [2]. The $u - s$ quark mass difference was determined from the $\Lambda^0 - N$ mass difference to be 127 MeV. Finally the matrix element P_{00}^{us} of the η exchange potential in u, s and d, s quark pair states was assumed to equal the matrix element of the K -exchange interaction: $P_{00}^{us} = P_{00}^K$. The two remaining matrix elements of the η exchange interaction for pair states

of light quarks P_{00}^{uu} and of s -quarks P_{00}^{ss} were determined from the matrix element P_{00}^{us} using the quark mass scaling relations

$$P_{nl}^{us} = \left(\frac{m_u}{m_s}\right)P_{nl}^{uu}, \quad P_{nl}^{ss} = \left(\frac{m_u}{m_s}\right)P_{nl}^{us}. \quad (3.1)$$

The constituent masses of the u and d quarks were taken to be equal and to be 340 MeV, and hence $m_s = 467$ MeV.

We shall treat the matrix elements of the charm (D) and strangeness-charm (D_s) exchange interaction potentials in (1.2) as phenomenological parameters, denoted $P_{00}^D = \langle 000|V_D(r)|000 \rangle$ and $P_{00}^{D_s} = \langle 000|V_{D_s}(r)|000 \rangle$ respectively. In view of the near degeneracy of the charm and the charm-strange D and D_s pseudoscalar mesons and of the corresponding D^* and D_s^* vector mesons we shall assume the matrix element equality

$$P_{00}^D = P_{00}^{D_s}. \quad (3.2)$$

It will be shown below that the empirical $\Sigma_c^* - \Sigma_c$ splitting indicates the magnitude of these matrix elements to be about 3 times smaller than that of the corresponding K exchange matrix element P_{00}^K .

The numerical predictions for the $C = 1$ ground state charm hyperon mass values are given in Table 1. The masses of all these states, with the exception of the Ω_C^{0*} , have now been determined experimentally [10,11,12]. In the Table we give the predicted mass values with and without the phenomenological D and D_s interaction matrix elements, which are required in the present model for lifting the degeneracy between the Σ_c and Σ_c^* and that between the Ξ_c^s and Ξ_c^* .

The magnitude of the matrix element P_{00}^D may be extracted from the empirical splitting between the Σ_c and the Λ_c^+ :

$$m(\Sigma_c) - m(\Lambda_c^+) = 8P_{00}^\pi - 4P_{00}^D - \frac{4}{3}P_{00}^{uu}. \quad (3.3)$$

This yields the value $P_{00}^D = 6.5$ MeV. It is worth noting that the ratio between this value and the corresponding value 20.1 MeV for the K exchange interaction matrix element P_{00}^K is close to the quark mass ratio m_s/m_c that

would be suggested by comparison of the expressions for the K and D exchange pseudoscalar exchange interactions, if the coupling strengths of these two interactions are equal. To see this we note that the value for the constituent mass of the s -quark was determined to be 467 MeV in [2]. The corresponding value for the c -quark may be determined from the difference between the Λ_c^+ and Λ^0 :

$$m_c = m(\Lambda_c^+) - m(\Lambda^0) + m_s + 6P_{00}^K - 6P_{00}^D - \frac{1}{2}\delta. \quad (3.4)$$

This yields the value $m_c = 1652$ MeV, when in the expression for the mass difference correction δ (2.5) we use the value $\hbar\omega = 157$ MeV [2]. These values for m_s and m_c are very close to those obtained in ref.[13]. The ratio $m_s/m_c = 0.28$, is then only slightly smaller than the matrix element ratio $P_{00}^D/P_{00}^K = 0.32$. The fact that the latter number is slightly larger is natural, as the relative importance of the vector meson exchange interaction should be larger in the case D and D^* exchange than in the case of K and K^* exchange in view of the near degeneracy of the D and D^* mesons. The near equality between the quark mass and matrix element ratios suggests that the overall interaction strength is approximately $SU(4)_F$ symmetric and that this flavor symmetry is broken mainly through the quark mass differences.

With the numerical value 6.5 MeV for P_{00}^D the Σ_c mass is fitted to be in agreement with the experimental value, whereas if the D exchange matrix element is set to 0, the mass of the Σ_c is overpredicted by 26 MeV. Since this represents only $\simeq 15\%$ of the mass splitting between the Σ_c and the Λ_c^+ it is clear that the dominant part of the hyperfine splitting of the hyperons with only one heavy quark is due to the hyperfine interaction between the two light quarks. This value for P_{00}^D is however not sufficient to explain all of the splitting between the Σ_c^* and the Σ_c , which is solely due the fine structure interaction between the light and the charm quarks. This splitting is obtained as $6P_{00}^D = 39$ MeV, which is considerably smaller than the empirical splitting of 75 MeV. As the present experimental mass value for the Σ_c^* remains very uncertain there is little reason at this time to increase the value for P_{00}^D to reduce this difference, most of which (60 MeV) could be accounted for by taking P_{00}^D to be 10 MeV, at a price of a concomitant (quite insignificant) underprediction of 14 MeV of the Σ_c .

The empirical splitting between the Ξ_c^a and the Σ_c is only 10 – 15 MeV. The expression for this splitting is in the present model

$$m(\Xi_c^a) - m(\Sigma_c) = P_{00}^\pi - 6P_{00}^K + \frac{1}{3}P_{00}^{uu} - 2P_{00}^{us} + 7P_{00}^D - 3P_{00}^{D_s} + \Delta_s. \quad (3.5)$$

With the matrix element values above this splitting is also predicted to be small: 30 MeV. The agreement with the empirical splitting may be improved by exploiting the at least 10 MeV large uncertainty in the value 127 MeV for the quark mass difference parameter Δ_s . Perfect agreement with the empirical splitting can of course be achieved by relaxing the assumed matrix element equality (3.2) and taking $P_{00}^{D_s}$ to be 11 MeV rather than 6.5 MeV. The smallness of the $\Xi_c^a - \Sigma_c$ splitting does in fact provide a rather sensitive test of the model, in view of the requirement of balancing the $u - s$ quark mass difference against the fine structure matrix elements.

The empirical splitting between the Ξ_c^s and the Ξ_c^a is 95 MeV, within a considerable uncertainty range. The present prediction for this splitting is

$$m(\Xi_c^s) - m(\Xi_c^a) = 4P_{00}^K + \frac{8}{3}P_{00}^{us} - 2P_{00}^D - 2P_{00}^{D_s}, \quad (3.6)$$

the numerical value of which is 108 MeV. This is only about 10% larger than the empirical splitting 90-95 MeV.

The splitting between the Ξ_c^* and the Ξ_c^s is predicted to be

$$m(\Xi_c^*) - m(\Xi_c^s) = 3P_{00}^D + 3P_{00}^{D_s}. \quad (3.7)$$

Numerically this is 39 MeV, which amounts to only about one half of the empirical splitting 82 MeV. Because of the substantial uncertainty in the empirical mass value for the Ξ_c^s the predicted value may nevertheless turn out to be satisfactory. The splitting can of course in principle be fully accounted for in the same way as the $\Sigma^* - \Sigma_c$ splitting by increasing the values of the matrix elements P_{00}^D and $P_{00}^{D_s}$.

The splitting between the Ω_c^0 and the Ξ_c^a is predicted to be

$$m(\Omega_c^0) - m(\Xi_c^a) = 6P_{00}^K + P_{00}^{us} - \frac{4}{3}P_{00}^{ss} + 3P_{00}^D + 3P_{00}^{D_s} + \Delta_s. \quad (3.8)$$

The numerical value for this splitting is 241 MeV, which agrees very well with the empirical value 245 MeV. Finally the $\Omega_c^{0*} - \Omega_c^0$ splitting is predicted to be $6P_{00}^{D_s} = 39$ MeV.

It is worth noting that the fine structure corrections in Table 1 imply the equal spacing rule [14]

$$m(\Sigma_c^*) - m(\Sigma_c) = m(\Xi_c^*) - m(\Xi_c^s) = m(\Omega_c^{0*}) - m(\Omega_c^0) \quad (3.9)$$

under the matrix element equality assumption (3.2). With the near equality of the present empirical values for the first two of these splittings (75 MeV, 82 MeV) this rule appears to be well satisfied. This equal spacing rule implies the weaker mass relation for these charm hyperons proposed in ref. [15].

It is interesting to note how similar the present predictions of the splittings of the ground state charm hyperons are to those obtained previously in the topological soliton (Skyrme) model, which implies an underlying large N_c limit. The $\Sigma_c - \Lambda_c^+$ splitting was predicted to be 154-170 MeV in ref. [16] and 179 MeV in ref. [17]. These values are close to the present predictions in Table 1 and to the empirical one. The quark model based prediction of the splitting between the Σ_c^* and the Σ_c above of ~ 39 MeV falls within the range 38-62 MeV predicted in ref. [16], but is larger than the value 25 MeV obtained in ref. [17].

4. The Negative Parity States with $L = 1$

The unperturbed harmonic oscillator energies for the lowest negative parity excitations with $L = 1$ have equal energy and lie $\hbar\omega$ above the unperturbed ground state. The corrections to this unperturbed level that arise from the flavor exchange interactions (1.1) and (1.2) are readily calculated using the methods of ref. [2], and are listed in Tables 2 and 3 for the Λ_c^+ , Σ_c and Ξ_c and Ω_c^0 respectively. In these expressions we also have included the correction that arises from mass difference perturbation (2.3) in lowest order. This perturbation is flavor dependent and takes the following values in the different P -shell multiplets:

$$\langle \Lambda_c^+ | H_0'' | \Lambda_c^+ \rangle_{[21]_{FS}[111]_F[21]_S} = -\frac{2}{3}\delta \quad (4.1a)$$

$$\langle \Lambda_c^+ | H_0'' | \Lambda_c^+ \rangle_{[21]_{FS}[21]_F[21]_S} = -\frac{2}{3}\delta \quad (4.1b)$$

$$\langle \Lambda_c^+ | H_0'' | \Lambda_c^+ \rangle_{[21]_{FS}[21]_F[3]_S} = -\frac{7}{12}\delta. \quad (4.1c)$$

The corresponding corrections for the P -shell excitations of the Σ_c are

$$\langle \Sigma_c^+ | H_0'' | \Sigma_c^+ \rangle_{[21]_{FS}[21]_F[21]_S} = -\frac{2}{3}\delta, \quad (4.2a)$$

$$\langle \Sigma_c^+ | H_0'' | \Sigma_c^+ \rangle_{[21]_{FS}[3]_F[21]_S} = -\frac{2}{3}\delta, \quad (4.2b)$$

$$\langle \Sigma_c^+ | H_0'' | \Sigma_c^+ \rangle_{[21]_{FS}[21]_F[3]_S} = -\frac{3}{4}\delta. \quad (4.2c)$$

Finally the corresponding corrections for the $L = 1$ negative parity excitations of the Ξ_c^a are the same as those of the Λ_c^+ (4.1), and those of the Ξ_c^b are the same as those of the Σ_c (4.2) (neglecting the difference between the u, d and s quarks in this correction).

In view of the short range of the heavy flavor exchange interaction it is natural to expect the P -shell matrix elements of this interaction to be small, as the corresponding oscillator wavefunctions vanish at short range. With $P_{11}^D = 0$ the predicted energy of the central of the lowest negative parity Λ_c^+ multiplet falls at 2599 MeV, which is only 10 MeV below the corresponding empirical value 2609 MeV. The latter value is obtained under the assumption that the two recently discovered $\Lambda_c^+(2593)$ and $\Lambda_c^+(2625)$ resonances form a negative parity spin doublet, which corresponds to the low lying $\Lambda(1405) - \Lambda(1520)$ strange flavor singlet spin doublet. The small underprediction of 10 MeV can in principle be removed by choosing $P_{11}^D = 5$ MeV. We shall accordingly employ this value in the numerical predictions below, although the smallness of this value makes the changes from the predictions that are obtained by setting it to 0 are in fact too small to be significant at the expected level of accuracy of the model.

The predicted energies of the other Λ_c^+ and Σ_c negative parity states in Table 2 are expected to be fairly realistic in view of the remarkably satisfactory prediction obtained for the centroid energy of the $\Lambda_c^+(2593) - \Lambda_c^+(2625)$ doublet. The present prediction for splitting between the ground state and this doublet is 25 - 50 MeV larger than the corresponding ones obtained in refs.[18,19], in which the fine structure interaction between the quarks was described in terms of one gluon exchange.

In Table 3 we list the predicted energies of the negative parity excited states of the Ξ_c and the Ω_c^0 hyperons with $L = 1$. We expect the reliability of these predictions to be similar to those obtained for the corresponding states of the Λ_c and Σ_c hyperons above.

Among the negative parity states listed in Tables 2 and 3 are notes that some are predicted to be near degenerate. Thus the Λ_c^+ multiplets with zero order wavefunctions with mixed flavor symmetry $[21]_F$ are predicted to be split by only 12 MeV. A similar near degeneracy is predicted for the $[21]_F \Xi_c^a$ with $S = 1/2$ and $[21]_F \Xi_c^s$ with $S = 3/2$. The $\frac{1}{2}^-$ and $\frac{3}{2}^-$ of these doublets may therefore be too close to be experimentally resolvable.

The present prediction of the position of the energy of the $\Lambda_c(2593)^+ - \Lambda_c(2625)^+$ negative parity doublet is the first that is in agreement with the empirical value. The quark model predictions of ref. [5] overpredicts the position by 70 MeV, and that of ref. [18] overpredicts the spin-orbit splitting of the multiplet by a large factor. The soliton model prediction of ref. [16] also overpredicts this spin-orbit splitting.

5. The Spectrum of the $B = -1$ Hyperons

The spectrum of the $B = -1$ hyperons that is predicted with the same model as that used above for the $C = +1$ hyperons will differ from the former in only two aspects. The first is that the larger constituent mass of the b -quark will increase the mass difference correction (2.5) slightly and the second is the fact that the generalization of the heavy flavor exchange in-

teraction (1.2) to the case of the bottom hyperons should be less important than for the charm hyperons in view of the very short range of bottom flavor exchange mechanisms and the smallness of the overall factor $1/m_b$ that is expected to be associated with the B -meson exchange interaction. The latter of these two features would imply that the splitting between the Σ_b and Σ_b^* , as well as $\Xi_b^{(s)}$ and Ξ_b^* and the Ω_b^- and Ω_b^{*-} states should be very small (no more than ~ 10 -20 MeV). The first data on the masses of the Σ_b and Σ_b^* hyperons does however give their mass splitting as 56 MeV, although within a large uncertainty limit [20]. This - if confirmed - indicates that B -exchange mechanisms cannot be neglected.

The expression for the mass difference between the Σ_b and Λ_b is, in analogy with (3.3),

$$m(\Sigma_b) - m(\Lambda_b) = 8P_{00}^\pi - 4P_{00}^B - \frac{4}{3}P_{00}^{uu}. \quad (5.1)$$

Here P_{00}^B is defined as the matrix element $\langle 000|V_B(r)|000 \rangle$, where $V_B(r)$ is the effective B -meson exchange interaction in the $SU(4)_F$ extension of the interaction (1.2). With the $\Sigma_b - \Lambda_b^0$ mass difference value 173 MeV [20] and using the same values for the matrix elements of the π and η exchange interactions we then obtain $P_{00}^B = 6$ MeV, which is almost as large as the corresponding value for the C -exchange matrix element P_{00}^D .

The value for the constituent mass of the b -quark may be determined from the mass splitting between the Λ_b^0 mass and the Λ^0 in analogy with (3.4) as

$$m_b = m(\Lambda_b^0) - m(\Lambda^0) + m_s + 6P_{00}^K - 6P_{00}^B - \frac{1}{2}\delta. \quad (5.2)$$

This gives $m_b = 5003$ MeV. Here we take $m_h = m_b$ in the expression (2.5) for the quark mass difference correction.

The predicted $B = -1$ hyperon states in the ground state band are listed in Table 4. The predicted $\Sigma_b^* - \Sigma_b$ splitting is $6P_{00}^B = 36$ MeV if P_{00}^B is taken to be 6 MeV as suggested by the empirical $\Sigma_b - \Lambda_b^0$ splitting. This splitting falls within the uncertainly limits of the empirical splitting 56 ± 22 [20]. The predicted value for the $\Xi_b^a - \Sigma_b$ splitting is

$$m(\Xi_b^a) - m(\Sigma_b) = P_{00}^\pi - 6P_{00}^K + \frac{1}{3}P_{00}^{uu} - 2P_{00}^{us} + 7P_{00}^B - 3P_{00}^{B_s} + \Delta_s. \quad (5.3)$$

If the matrix element of the B_s exchange interaction $P_{00}^{B_s} = \langle 000 | V_{B_s}(r) | 000 \rangle$ is taken to be equal to that of the B -exchange interaction (6 MeV) this splitting comes out as 28 MeV. The splitting between the $B = -1$ cascade hyperons Ξ_b^s and Ξ_b^a is

$$m(\Xi_b^s) - m(\Xi_b^a) = 4P_{00}^K + \frac{8}{3}P_{00}^{uu} - 2P_{00}^B - 2P_{00}^{B_s}. \quad (5.4)$$

Using the same matrix element values as above, this splitting is predicted to be 110 MeV.

The splitting between the Ξ_b^* and the Ξ_b^s is predicted (in analogy with (3.7)) to be $3P_{00}^B + 3P_{00}^{B_s} = 36$ MeV. The splitting between Ω_b^- and the Ξ_b^a is

$$m(\Omega_b^-) - m(\Xi_b^a) = 6P_{00}^K + P_{00}^{us} - \frac{4}{3}P_{00}^{ss} + 3P_{00}^B + 3P_{00}^{B_s} + \Delta_s. \quad (5.5)$$

The predicted numerical value for this splitting is 238 MeV - i.e. it should be almost equal to that between the Ω_c^0 and the Ξ_c^a . Finally the $\Omega_b^{*-} - \Omega_b^-$ splitting is predicted to be $6P_{00}^{B_s} \simeq 36$ MeV. The predicted mass values in Table 4 are close to those obtained in ref. [13], once the latter are shifted up by the 20 MeV needed to bring the predicted mass of the Λ_b into agreement with its empirical value.

The size of the empirical splitting between the Σ_b^* and the Σ_b is only 2 times smaller than that between the Σ_c^* and the Σ_c . This is larger than the ratio $\simeq 1/3$ that would be suggested by both the present pseudoscalar exchange model, and the gluon exchange model for the hyperfine interaction. The empirical splitting is also much larger than the prediction of the bound state version of the Skyrme model [16,17].

In Table 5 we list the predicted negative parity excitations with $L = 1$ of the $B = -1$ hyperons. In these predictions we have not included any B -meson exchange interactions, as the contribution from such are expected

to be smaller than the uncertainty range of the predictions obtained with the present model and as in the absence of empirical data on the energies of the $B = -1$ hyperon resonances in the $N = 1$ band the p-state matrix elements of the $B-$ and B_s -exchange interactions are unknown. Because of the neglect of bottom exchange interactions the $S = 1/2$ multiplets within the different quark flavor combinations are predicted to be degenerate.

The predicted central position of the lowest Λ_b^0 negative parity multiplet in Table 5 is 300 MeV above the Λ_b^0 . This is ~ 30 MeV below the corresponding predictions obtained in ref. [5], where the fine structure interaction between the quarks was described in terms of gluon exchange. The Skyrme model predictions of ref. [16] for this central position are 30-50 MeV lower.

6. Magnetic Moments of the Heavy Flavor Hyperons

The expressions for the magnetic moments of the ground state charm hyperons in the constituent quark model in the impulse approximation have been derived in ref. [21]. These expressions, which are linear combinations of ratios of the nucleon and relevant constituent quark masses, are listed in Table 6. In the table the corresponding numerical values are also listed as obtained with the constituent mass values used above (i.e. $m_u = 340$ MeV, $m_s = 467$ MeV, $m_c = 1652$ MeV).

A flavor dependent interaction between the quarks of the form (1.1) or (1.2) implies charge exchange between quarks of unequal charge, and hence also of two-body or exchange current operators [2,22]. The general form of the octet vector exchange current operator that is associated with the pseudoscalar octet mediated interaction (1.1) will have the form [2]:

$$\begin{aligned} \vec{\mu}^{\pi K} = \mu_N \{ & \tilde{V}_\pi(r_{ij})(\vec{\tau}_i \times \vec{\tau}_j)_3 \\ & + \tilde{V}_K(r_{ij})(\lambda_i^4 \lambda_j^5 - \lambda_j^4 \lambda_i^5) \} (\vec{\sigma}_i \times \vec{\sigma}_j). \end{aligned} \quad (6.1)$$

Here $\tilde{V}_\pi(r)$ and $\tilde{V}_K(r)$ are dimensionless functions, which describe the spatial structure of the π and K exchange magnetic moment operators and μ_N is

the nuclear magneton.

The charm exchange interaction (6.2) will give rise to a similar exchange current operator that involves $SU(4)_F$ matrices with the form

$$\begin{aligned}\vec{\mu}^{DD_s} = \mu_N \{ & \tilde{V}_D(r_{ij})(\lambda_i^{11}\lambda_j^{12} - \lambda_i^{12}\lambda_j^{11}) \\ & + \tilde{V}_{D_s}(r_{ij})(\lambda_i^{13}\lambda_j^{14} - \lambda_i^{14}\lambda_j^{13}) \} (\vec{\sigma}_i \times \vec{\sigma}_j)\end{aligned}\quad (6.2)$$

Here $\tilde{V}_D(r_{ij})$ and $\tilde{V}_{D_s}(r_{ij})$ are then the corresponding dimensionless functions, which describe the spatial structure of the D (or D and D^*) and D_s (or D_s and D_s^*) exchange magnetic moments.

The exchange corrections to the magnetic moments of the hyperons in the ground state band will only depend on the S -shell matrix elements of the spatial functions $\tilde{V}_a(r)$ in (6.1) and (6.2) ($a = \pi, K, D, D_s$).

The expressions for these exchange current corrections to the magnetic moments of $J = \frac{1}{2}$ the charm hyperons are then

$$\begin{aligned}\mu^{ex}(\Lambda_c^+) &= -\mu^{ex}(\Sigma_c^+) = -\frac{1}{2}\mu^{ex}(\Sigma_c^0) \\ &= 2 < \varphi_{000}(\vec{r}_{12}) | \tilde{V}_D(r_{12}) | \varphi_{000}(\vec{r}_{12}) > \mu_N,\end{aligned}\quad (6.3a)$$

$$\begin{aligned}\mu^{ex}(\Xi_c^{a+}) &= -\mu^{ex}(\Xi_c^{s+}) = -\mu^{ex}(\Omega_c^0) = \\ &= 2 < \varphi_{000}(\vec{r}_{12}) | \tilde{V}_{D_s}(r_{12}) | \varphi_{000}(\vec{r}_{12}) > \mu_N,\end{aligned}\quad (6.3b)$$

$$\mu^{ex}(\Xi_c^{a0}) = -\mu^{ex}(\Xi_c^{s0}) = \mu^{ex}(\Lambda_c^+) + \mu^{ex}(\Xi_c^+), \quad (6.3c)$$

$$\mu^{ex}(\Sigma_c^{++}) = 0. \quad (6.3d)$$

The exchange current corrections to the corresponding transition magnetic moments are

$$\mu^{ex}(\Sigma_c^+ \rightarrow \Lambda_c^+) = \frac{2}{\sqrt{3}} < \varphi_{000}(\vec{r}_{12}) | 2\tilde{V}_\pi(r_{12}) - \tilde{V}_D(r_{12}) | \varphi_{000}(\vec{r}_{12}) >, \quad (6.4a)$$

$$\mu^{ex}(\Xi_c^{s+} \rightarrow \Xi_c^{a+}) = \frac{2}{\sqrt{3}} < \varphi_{000}(\vec{r}_{12}) | 2\tilde{V}_K(r_{12}) - \tilde{V}_{D_s}(r_{12}) | \varphi_{000}(\vec{r}_{12}) >, \quad (6.4b)$$

$$\mu^{ex}(\Xi_c^{s0} \rightarrow \Xi_c^{a0}) = \frac{2}{\sqrt{3}} < \varphi_{000}(\vec{r}_{12}) | \tilde{V}_D(r_{12}) - \tilde{V}_{D_s}(r_{12}) | \varphi_{000}(\vec{r}_{12}) > . \quad (6.4c)$$

The matrix elements of the pion and kaon exchange current operators were treated completely phenomenologically in ref. [2], and judged to be small: $< \varphi_{000}(\vec{r}_{12}) | \tilde{V}_\pi(r_{12}) | \varphi_{000}(\vec{r}_{12}) > \simeq -0.02$, $< \varphi_{000}(\vec{r}_{12}) | \tilde{V}_K(r_{12}) | \varphi_{000}(\vec{r}_{12}) > \simeq 0.03$. It is natural to expect the corresponding D and D_s exchange current operators (6.2) to have even smaller matrix elements, thus rendering this exchange current contribution insignificant. This implies that it is only the transition magnetic moments (6.4) that may have significant exchange current contributions, as it is only these that contain terms associated with pion and kaon exchange. In Table 6 we have therefore only included those exchange current corrections (in column II). These exchange current contributions are not large enough to affect the quark model predictions of the magnetic moments of the charm hyperons in any significant way.

Without any D or D_s exchange current contributions the magnetic moments of the Λ_c^+ , Ξ_c^{a+} and Ξ_c^{a0} are predicted to be equal, the numerical value as determined by the quark masses being $0.38\mu_N$. It is remarkable that this value almost completely coincides with the corresponding average value $0.37\mu_N$, which is given by the leading pion loop contribution in chiral perturbation theory [23]. If the unknown constant in the chiral perturbation theory calculation in ref.[23] is dropped the magnetic moment of the Ξ_c^{a+} is $0.42\mu_N$ and that of the Ξ_c^{a0} is $0.37\mu_N$. Such a deviation of the charm cascade magnetic moments from the average value can in the present approach be understood if the D - and D_s -meson exchange current magnetic moment matrix elements in (6.3) are negative, and the former is larger in magnitude: e.g. $< 000 | \tilde{V}_D(r) | 000 > \sim -0.04$ and $< 000 | \tilde{V}_{D_s}(r) | 000 > \sim -0.02$. The presence and possible significance of charm exchange current corrections can naturally only be decided by empirical determination of the magnetic moments of the charm hyperons. Note that in the bound state approach to the Skyrme model the magnetic moments of the Λ_c^+ and the Ξ_c^{a+} , Ξ_c^{a0} are also predicted to be degenerate [23].

In the case of the $B = -1$ hyperons the exchange current contributions should be very small, with the exception of the transition moments, which again also obtain a pion exchange contribution. In the case of the Λ_b and the Σ_b these exchange current contributions can be obtained from the cor-

responding expressions (10.3) in ref. [2], for the strange hyperons, dropping the kaon exchange contribution (or by replacing it by a B -meson exchange contribution of the same form).

The impulse approximation expressions for the $B = -1$ hyperons can be obtained from the corresponding expressions for the strange hyperons, by substituting the b -quark mass in place of that of the S -quark. For the Λ_b^0 and Σ_b hyperons we obtain, using the quark mass values $m_u = 340$ MeV and $m_b = 5039$:

$$\mu(\Lambda_b^0) = \mu(\Xi_b^{a0}) = \mu(\Xi_b^{a-}) = -\frac{1}{3} \frac{m_N}{m_b} = -0.062\mu_N, \quad (6.5a)$$

$$\mu(\Sigma_b^+) = \frac{8}{9} \frac{m_N}{m_u} + \frac{1}{9} \frac{m_N}{m_b} = 2.47\mu_N \quad (6.5b)$$

$$\mu(\Sigma_b^0) = \frac{2}{9} \frac{m_N}{m_u} + \frac{1}{9} \frac{m_N}{m_b} = 0.63\mu_N \quad (6.5c)$$

$$\mu(\Sigma_b^-) = \mu(\Omega_b^-) = -\frac{4}{9} \frac{m_N}{m_u} + \frac{1}{9} \frac{m_N}{m_b} = -1.21. \quad (6.5d)$$

The smallness of the predicted value of the magnetic moment of the Λ_b^0 indicates that it will be difficult to measure accurately.

7. Discussion

The results presented here show that a quite satisfactory description of the presently known part of the spectra of the heavy flavor hyperons can be obtained by describing the fine structure interaction between the quarks in terms of the schematic chiral field flavor-spin interaction (1.1), which represents the most important component of the interaction that is mediated by the $SU(3)_F$ octet of light pseudoscalar mesons. In order to obtain a non-vanishing splitting between the $S = 1/2$ and $S = 3/2$ $C = 1$ and $B = -1$ hyperons with zero order wavefunctions with $[21]_F$ (112) and $[3]_F$ (111) flavor symmetry, which is the analog of the octet decuplet splitting of the light and strange baryons, a corresponding phenomenological D -meson (or D -meson like) exchange interaction (1.2) also had to be included.

No attempt has been made here to explain the 32 MeV spin-orbit splitting between the $\Lambda_c(2593)^+$ and $\Lambda_c(2625)^+$. It is however interesting to note that this splitting is smaller by a factor 3.59 than the spin-orbit splitting of 115 MeV between the corresponding strange hyperon doublet $\Lambda(1405) - \Lambda(1520)$, because this factor coincides almost completely with the ratio between the constituent masses of the charm and strange hyperons $m_c/m_s = 3.54$ (using the present values for the quark masses). This strongly supports the view that these resonances are 3-quark states, that are split by a two-body spin-orbit interaction, which would be expected to be inversely proportional to the quark masses. The dynamical origin of such a spin-orbit interaction is expected to be a combination of the spin-orbit interaction that is associated with the effective confining interaction and vector meson and vector meson like multimeson exchange mechanisms [2].

The results presented above underpredict, as do - although to a lesser extent - those based on chiral perturbation theory in ref.[14], the approximately equal $\Sigma_c^* - \Sigma_c$ and $\Xi_c^* - \Xi_c^s$ splittings of 75 MeV and 82 MeV respectively. The present prediction of 39 MeV for this splitting can of course be increased to 50 - 60 MeV by relaxing the assumption of the matrix element equality (3.2) for the D and D_s exchange interaction, and the requirement of exact reproduction of the empirical $\Sigma_c - \Lambda_c^+$ and $\Xi_c^a - \Sigma_c$ splittings. The result then would agree well with the QCD estimate that the splitting should be

approximately $\Lambda_{QCD}^2/m_c \simeq 50$ MeV [14]. If the large uncertainty limit on the preliminary experimental result for the $\Sigma_b^* - \Sigma_b$ splitting [20] is taken into account, it also approximately agrees with this quark mass scaling rule. It would be important that the present large empirical uncertainty limits on the masses of the spin $\frac{3}{2}$, $C = 1$ hyperons be narrowed in order to assess whether or not the present underprediction of the value of the $\Sigma_c^* - \Sigma_c$ and $\Xi_c^* - \Xi_c^s$ splittings is a problem for the present chiral quark model. The overall quality of the predicted masses of the charm hyperons in the ground state band is quite satisfactory, the predictions being similar to those obtained in other recent work using different theoretical approaches [14,23]. The quark model with the chiral field interaction (1.1) between the light constituent quarks is the only one, however, that at the present time can describe the lowest negative parity resonances of the Λ_c in a quantitatively satisfactory way.

The main conclusion of the present work is that, with the exception of the splitting between the spin $\frac{1}{2}$ and $\frac{3}{2}$ states in the ground state band, the chiral field interaction (1.1) between the light constituent quarks is able to explain the empirically known part of the spectra of the $C = 1$ hyperons. It is natural to expect the splitting between the spin $\frac{1}{2}$ and $\frac{3}{2}$ states in the ground state band to reveal short range dynamics, which is absent or unimportant in the light and strange baryons. This splitting may also be used to settle question of the relative (un)importance of the gluon exchange interaction between heavy quarks and for pairs of light and heavy constituent quarks. The achievement of a deeper understanding of the nature and the dynamical origin of this splitting would be greatly facilitated by empirical determination of the ground state spectrum of the $C = 2$ hyperons, as the fine structure splitting of these should be entirely due to such short range dynamics.

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Table 1

Contributions to the masses (in MeV) of the $C = +1$ ground state hyperons from flavor exchange interactions (1.1) and (1.2) (δM). The difference between constituent masses of the u and s quarks is denoted Δ_s . The predicted mass values in column I are obtained without inclusion of the contribution from charm exchange mechanisms. The superscripts s, a on the Ξ_c states indicate that the light and strange quarks are in symmetric and antisymmetric states respectively. The experimental values are from refs.[10,11,12].

$[f]_{FS}[f]_F[f]_S$	State (mass)	Predicted mass I	Predicted mass II	δM
$[3]_{FS}[21]_F[21]_S$	Λ_c^+ (2285)	2285 (input)	2285 (input)	$-9P_{00}^\pi + P_{00}^{uu} - 6P_{00}^D$ $-\frac{1}{2}\delta$
$[3]_{FS}[21]_F[21]_S$	Σ_c^+ (2455?)	2481	2455 (input)	$-P_{00}^\pi - \frac{1}{3}P_{00}^{uu} - 10P_{00}^D$ $-\frac{1}{2}\delta$
$[3]_{FS}[3]_F[3]_S$	Σ_c^* (2530?)	2481	2494	$-P_{00}^\pi - \frac{1}{3}P_{00}^{uu} - 4P_{00}^D$ $-\frac{1}{2}\delta$
$[3]_{FS}[21]_F[21]_S$	$\Xi_c^{(a)}$ (2465-2470)	2485	2485	$-6P_{00}^K - 2P_{00}^{us}$ $-3P_{00}^D - 3P_{00}^{D_s}$ $+\Delta_s - \frac{1}{2}\delta$
$[3]_{FS}[21]_F[21]_S$	$\Xi_c^{(s)}$ (2560?)	2618	2593	$-2P_{00}^K + \frac{2}{3}P_{00}^{us}$ $-5P_{00}^D - 5P_{00}^{D_s}$ $+\Delta_s - \frac{1}{2}\delta$
$[3]_{FS}[3]_F[3]_S$	Ξ_c^* (2642?)	2618	2632	$-2P_{00}^K + \frac{2}{3}P_{00}^{us}$ $-2P_{00}^D - 2P_{00}^{D_s}$ $+\Delta_s - \frac{1}{2}\delta$
$[3]_{FS}[21]_F[21]_s$	Ω_c^0 (2710?)	2748	2726	$-\frac{4}{3}P_{00}^{ss} - 10P_{00}^{D_s}$ $+2\Delta_s - \frac{1}{2}\delta$
$[3]_{FS}[3]_F[3]_S$	Ω_c^{0*}	2748	2765	$-\frac{4}{3}P_{00}^{ss} - 4P_{00}^{D_s}$ $+2\Delta_s - \frac{1}{2}\delta$

Table 2

The negative parity $L = 1$ Λ_c^+ and Σ_c resonances as predicted in the harmonic oscillator model with the flavor exchange fine structure interactions (1.1) and (1.2). The fine structure corrections include the $u - s$ quark mass difference Δ_s and the corrections δ due to the mass difference between the light and heavy quarks (4.1). The predicted energies (in MeV) are given in brackets. The predicted value for the lowest Λ_c^+ doublet in square brackets is obtained with $P_{11}^D = 0$.

$[f]_{FS}[f]_F[f]_S$	Multiplet	average energy	δM
$[21]_{FS}[111]_F[21]_S$	$\frac{1}{2}^-, \Lambda_c(2593)^{+?}$ $\frac{3}{2}^-, \Lambda_c(2625)^{+?}$	2609 (2609) [2599]	$-\frac{9}{2}P_{00}^\pi + \frac{1}{2}P_{00}^{uu} - 6P_{00}^D$ $+\frac{3}{2}P_{11}^\pi - \frac{1}{6}P_{11}^{uu} + 2P_{11}^D$ $-\frac{2}{3}\delta$
$[21]_{FS}[21]_F[21]_S$	$\frac{1}{2}^-, \Lambda_c^+;$ $\frac{3}{2}^-, \Lambda_c^+$? (2643)	$-\frac{9}{2}P_{00}^\pi + \frac{1}{2}P_{00}^{uu} - 3P_{00}^D$ $+\frac{3}{2}P_{11}^\pi - \frac{1}{6}P_{11}^{uu} + 5P_{11}^D$ $-\frac{2}{3}\delta$
$[21]_{FS}[21]_F[3]_S$	$\frac{1}{2}^-, \Lambda_c^+;$ $\frac{3}{2}^-, \Lambda_c^+;$ $\frac{5}{2}^-, \Lambda_c^+$? (2655)	$-3P_{00}^D$ $+3P_{11}^\pi - \frac{1}{3}P_{11}^{uu} + P_{11}^D$ $-\frac{7}{12}\delta$
$[21]_{FS}[21]_F[21]_S$	$\frac{1}{2}^-, \Sigma_c;$ $\frac{3}{2}^-, \Sigma_c$? (2747)	$-\frac{1}{2}P_{00}^\pi - \frac{1}{6}P_{00}^{uu} - 5P_{00}^D$ $+\frac{3}{2}P_{11}^\pi + \frac{1}{2}P_{11}^{uu} + 3P_{11}^D$ $-\frac{2}{3}\delta$
$[21]_{FS}[3]_F[21]_S$	$\frac{1}{2}^-, \Sigma_c;$ $\frac{3}{2}^-, \Sigma_c$? (2693)	$-\frac{1}{2}P_{00}^\pi - \frac{1}{6}P_{00}^{uu} - 2P_{00}^D$ $+\frac{3}{2}P_{11}^\pi + \frac{1}{2}P_{11}^{uu} + 6P_{11}^{DD*}$ $-\frac{2}{3}\delta$
$[21]_{FS}[21]_F[3]_S$	$\frac{1}{2}^-, \Sigma_c;$ $\frac{3}{2}^-, \Sigma_c;$ $\frac{5}{2}^-, \Sigma_c$? (2654)	$-P_{00}^\pi - \frac{1}{3}P_{00}^{uu} - P_{00}^D$ $+3P_{11}^D$ $-\frac{3}{4}\delta$

Table 3

The negative parity $L = 1$ Ξ_c and Ω_c^0 resonances predicted with the flavor exchange fine structure interactions (1.1) and (1.2). The quark mass difference corrections Δ_s and δ are indicated explicitly. The energies are given in units of MeV.

$[f]_{FS}[f]_F[f]_S$	Multiplet	average energy	δM
$[21]_{FS}[111]_F[21]_S$	$\frac{1}{2}^-, \Xi_c^a,$ $\frac{3}{2}^-, \Xi_c^a$	2752	$-P_{00}^{us} - 3P_{00}^K - 3P_{00}^D - 3P_{00}^{D_s}$ $+ \frac{1}{3}P_{11}^{us} + P_{11}^K + P_{11}^D + P_{11}^{D_s}$ $+ \Delta_s - \frac{2}{3}\delta$
$[21]_{FS}[21]_F[21]_S$	$\frac{1}{2}^-, \Xi_c^a,$ $\frac{3}{2}^-, \Xi_c^a$	2787	$-P_{00}^{us} - 3P_{00}^K - \frac{3}{2}P_{00}^D - \frac{3}{2}P_{00}^{D_s}$ $+ \frac{1}{3}P_{00}^{us} + P_{11}^K + \frac{5}{2}P_{11}^D + \frac{5}{2}P_{11}^{D_s}$ $+ \Delta_s - \frac{2}{3}\delta$
$[21]_{FS}[21]_F[3]_S$	$\frac{1}{2}^-, \Xi_c^a,$ $\frac{3}{2}^-, \Xi_c^a,$ $\frac{5}{2}^-, \Xi_c^a$	2898	$-\frac{3}{2}P_{00}^D - \frac{3}{2}P_{00}^{D_s}$ $+ \frac{2}{3}P_{11}^{us} + 2P_{11}^K + \frac{1}{2}P_{11}^D + \frac{1}{2}P_{11}^{D_s}$ $+ \Delta_s - \frac{7}{12}\delta$
$[21]_{FS}[21]_F[21]_S$	$\frac{1}{2}^-, \Xi_c^s,$ $\frac{3}{2}^-, \Xi_c^s$	2851	$\frac{1}{3}P_{00}^{us} - P_{00}^K - \frac{5}{2}P_{00}^D - \frac{5}{2}P_{00}^{D_s}$ $3P_{11}^K - P_{11}^{us} + \frac{3}{2}P_{11}^D + \frac{3}{2}P_{11}^{D_s}$ $+ \Delta_s - \frac{2}{3}\delta$
$[21]_{FS}[3]_F[21]_S$	$\frac{1}{2}^-, \Xi_c^s,$ $\frac{3}{2}^-, \Xi_c^s$	2886	$\frac{1}{3}P_{00}^{us} - P_{00}^K - P_{00}^D - P_{00}^{D_s}$ $- P_{11}^{us} + 3P_{11}^K + 3P_{11}^D + 3P_{11}^{D_s}$ $+ \Delta_s - \frac{2}{3}\delta$
$[21]_{FS}[21]_F[3]_S$	$\frac{1}{2}^-, \Xi_c^s,$ $\frac{3}{2}^-, \Xi_c^s,$ $\frac{5}{2}^-, \Xi_c^s$	2792	$\frac{2}{3}P_{00}^{us} - 2P_{00}^K - \frac{1}{2}P_{00}^D - \frac{1}{2}P_{00}^{D_s}$ $+ \frac{3}{2}P_{11}^D + \frac{3}{2}P_{11}^{D_s}$ $+ \Delta_s - \frac{3}{4}\delta$
$[21]_{FS}[21]_F[21]_S$	$\frac{1}{2}^-, \Omega_c^0,$ $\frac{3}{2}^-, \Omega_c^0$	2965	$-\frac{2}{3}P_{00}^{ss} - 5P_{00}^{D_s}$ $+ 2P_{11}^{ss} + 3P_{11}^{D_s}$ $+ 2\Delta_s - \frac{2}{3}\delta$
$[21]_{FS}[3]_F[21]_S$	$\frac{1}{2}^-, \Omega_c^0,$ $\frac{3}{2}^-, \Omega_c^0$	2999	$-\frac{2}{3}P_{00}^{ss} - 2P_{00}^{D_s}$ $+ 2P_{11}^{ss} + 6P_{11}^{D_s}$ $+ 2\Delta_s - \frac{2}{3}\delta$
$[21]_{FS}[21]_F[3]_S$	$\frac{1}{2}^-, \Omega_c^0,$ $\frac{3}{2}^-, \Omega_c^0,$ $\frac{5}{2}^-, \Omega_c^0$	2936	$-\frac{4}{3}P_{00}^{ss} - P_{00}^{D_s}$ $+ 3P_{11}^{D_s}$ $+ 2\Delta_s - \frac{3}{4}\delta$

Table 4

Contributions to the masses (in MeV) of the $B = -1$ ground state hyperons from flavor exchange interactions (1.1) and (1.2) (δM). The difference between constituent masses of the u and s quarks is denoted Δ_s . The predicted mass values are given in brackets. The superscripts s, a on the Ξ_b states indicate that the light and strange quarks are in symmetric and anti-symmetric states respectively. The empirical values are from refs. [10,20].

$[f]_{FX}[f]_S$	State (mass)	Mass	δM
$[3]_{FS}[21]_F[21]_S$	Λ_b^0	5641 (input)	$-9P_{00}^\pi + P_{00}^{uu} - 6P_{00}^B$ $-\frac{1}{2}\delta$
$[3]_{FS}[21]_F[21]_S$	Σ_b	5814 (input)	$-P_{00}^\pi - \frac{1}{3}P_{00}^{uu} - 10P_{00}^B$ $-\frac{1}{2}\delta$
$[3]_{FS}[3]_F[3]_S$	Σ_b^*	5870 (5850)	$-P_{00}^\pi - \frac{1}{3}P_{00}^{uu} - 4P_{00}^B$ $-\frac{1}{2}\delta$
$[3]_{FS}[21]_F[21]_S$	$\Xi_b^{(a)}$? (5842)	$-6P_{00}^K - 2P_{00}^{us}$ $-3P_{00}^B - 3P_{00}^{B_s}$ $+\Delta_s - \frac{1}{2}\delta$
$[3]_{FS}[21]_F[21]_S$	$\Xi_b^{(s)}$? (5952)	$-2P_{00}^K + \frac{2}{3}P_{00}^{us}$ $-5P_{00}^B - 5P_{00}^{B_s}$ $+\Delta_s - \frac{1}{2}\delta$
$[3]_{FS}[3]_F[3]_S$	Ξ_b^*	? (5988)	$-2P_{00}^K + \frac{2}{3}P_{00}^{us}$ $-2P_{00}^B - 2P_{00}^{B_s}$ $+\Delta_s - \frac{1}{2}\delta$
$[3]_{FS}[21]_F[21]_S$	Ω_b^0	? (6080)	$-\frac{4}{3}P_{00}^{ss} - 10P_{00}^{B_s}$ $+2\Delta_s - \frac{1}{2}\delta$
$[3]_{FS}[3]_F[3]_S$	Ω_b^{0*}	? (6116)	$-\frac{4}{3}P_{00}^{ss} - 4P_{00}^{B_s}$ $+2\Delta_s - \frac{1}{2}\delta$

Table 5

The negative parity $L = 1$ $B = -1$ hyperon resonance energies (in MeV) as predicted in the harmonic oscillator model with the flavor exchange fine structure interactions (1.1). The fine structure corrections include the $u - s$ quark mass difference Δ_s and the corrections δ due to the mass difference between the light and heavy quarks (4.1). The neglect of the corrections of the $B-$ and B_s- exchange interactions implies a 20–30 MeV uncertainty in the predicted energies.

$[f]_{FS}[f]_F[f]_S$	Multiplet	Predicted energy	δM
$[21]_{FS}[111]_F[21]_S$ $[21]_{FS}[21]_F[21]_S$	$\frac{1}{2}^-, \Lambda_b^0;$ $\frac{3}{2}^-, \Lambda_b^0$	5940	$-\frac{9}{2}P_{00}^\pi + \frac{1}{2}P_{00}^{uu} + \frac{3}{2}P_{11}^\pi$ $-\frac{1}{6}P_{11}^{uu} - \frac{2}{3}\delta$
$[21]_{FS}[21]_F[3]_S$	$\frac{1}{2}^-, \Lambda_b^0; \frac{3}{2}^-, \Lambda_b^0;$ $\frac{5}{2}^-, \Lambda_b^0;$	6150	$+3P_{11}^\pi - \frac{1}{3}P_{11}^{uu}$ $-\frac{7}{12}\delta$
$[21]_{FS}[21]_F[21]_S$ $[21]_{FS}[3]_F[21]_S$	$\frac{1}{2}^-, \Sigma_b;$ $\frac{3}{2}^-, \Sigma_b$	6070	$-\frac{1}{2}P_{00}^\pi - \frac{1}{6}P_{00}^{uu} + \frac{3}{2}P_{11}^\pi$ $+\frac{1}{2}P_{11}^{uu} - \frac{2}{3}\delta$
$[21]_{FS}[21]_F[3]_S$	$\frac{1}{2}^-, \Sigma_b; \frac{3}{2}^-, \Sigma_b;$ $\frac{5}{2}^-, \Sigma_b;$	5950	$-P_{00}^\pi - \frac{1}{3}P_{00}^{uu}$ $-\frac{3}{4}\delta$
$[21]_{FS}[111]_F[21]_S$ $[21]_{FS}[21]_F[21]_S$	$\frac{1}{2}^-, \Xi_b^a;$ $\frac{3}{2}^-, \Xi_b^a$	6110	$-P_{00}^{us} - 3P_{00}^K + P_{11}^K$ $+\frac{1}{3}P_{11}^{us} + \Delta_s - \frac{2}{3}\delta$
$[21]_{FS}[21]_F[3]_S$	$\frac{1}{2}^-, \Xi_b^a; \frac{3}{2}^-, \Xi_c^a;$ $\frac{5}{2}^-, \Xi_b^a;$	6240	$+\frac{2}{3}P_{11}^{us} + 2P_{11}^K$ $+\Delta_s - \frac{7}{12}\delta$
$[21]_{FS}[21]_F[21]_S$ $[21]_{FS}[3]_F[21]_S$	$\frac{1}{2}^-, \Xi_b^s;$ $\frac{3}{2}^-, \Xi_b^s$	6200	$\frac{1}{3}P_{00}^{us} - P_{00}^K + 3P_{11}^K$ $-P_{11}^{us} + \Delta_s - \frac{2}{3}\delta$
$[21]_{FS}[21]_F[3]_S$	$\frac{1}{2}^-, \Xi_b^s; \frac{3}{2}^-, \Xi_c^s;$ $\frac{5}{2}^-, \Xi_b^s$	6120	$\frac{2}{3}P_{00}^{us} - 2P_{00}^K$ $+\Delta_s - \frac{3}{4}\delta$
$[21]_{FS}[21]_F[21]_S$ $[21]_{FS}[3]_F[21]_S$	$\frac{1}{2}^-, \Omega_b^-;$ $\frac{3}{2}^-, \Omega_b^-$	6310	$-\frac{2}{3}P_{00}^{ss} + 2P_{11}^{ss}$ $+2\Delta_s - \frac{2}{3}\delta$
$[21]_{FS}[21]_F[3]_S$	$\frac{1}{2}^-, \Omega_b^-; \frac{3}{2}^-, \Omega_b^-;$ $\frac{5}{2}^-, \Omega_b^-$	6240	$-\frac{4}{3}P_{00}^{ss}$ $+2\Delta_s - \frac{3}{4}\delta$

Table 6

Magnetic moments of the $S = 1/2$ $C = +1$ charm hyperons in the quark model (in units of the nuclear magneton). Column IA contains the impulse approximation expressions and column I the corresponding numerical values. Column II contains the exchange current corrections and column III the combined net prediction.

	IA	I	II	III
Λ_c^+	$\frac{2}{3} \frac{m_N}{m_c}$	0.38		0.38
Σ_c^{++}	$\frac{8}{9} \frac{m_N}{m_c} - \frac{2}{9} \frac{m_N}{m_c}$	2.33		2.33
Σ_c^+	$\frac{2}{9} \frac{m_N}{m_u} - \frac{2}{9} \frac{m_N}{m_c}$	0.49		0.49
Σ_c^0	$-\frac{4}{9} \frac{m_N}{m_u} - \frac{2}{9} \frac{m_N}{m_c}$	-1.35		-1.35
$\Sigma_c^+ \rightarrow \Lambda_c^+$	$-\frac{1}{\sqrt{3}} \frac{m_N}{m_u}$	-1.59	-0.04	-1.63
Ξ_c^{a+}	$\frac{2}{3} \frac{m_N}{m_c}$	0.38		0.38
Ξ_c^{a0}	$\frac{2}{3} \frac{m_N}{m_c}$	0.38		0.38
Ξ_c^{s+}	$\frac{4}{9} \frac{m_N}{m_u} - \frac{2}{9} \frac{m_N}{m_s} - \frac{2}{9} \frac{m_N}{m_c}$	0.65		0.65
Ξ_c^{s0}	$-\frac{2}{9} \frac{m_N}{m_u} - \frac{2}{9} \frac{m_N}{m_s} - \frac{2}{9} \frac{m_N}{m_c}$	-1.18		-1.18
$\Xi_c^{s+} \rightarrow \Xi_c^{a+}$	$-\frac{1}{\sqrt{3}} (2 \frac{m_N}{m_u} + \frac{m_N}{m_s})$	-1.45	0.07	-1.38
$\Xi_c^{s0} \rightarrow \Xi_c^{a0}$	$\frac{1}{3\sqrt{3}} (\frac{m_N}{m_u} - \frac{m_N}{m_s})$	0.14		0.14
Ω_c^0	$-\frac{4}{9} \frac{m_N}{m_s} - \frac{2}{9} \frac{m_N}{m_c}$	-1.02		-1.02